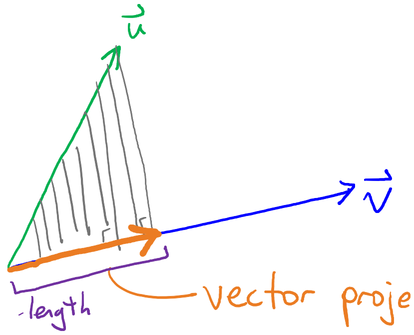
Given two vectors **u** and **v**, we might want to find the **component** of **u** in the direction of **v**:

$$\text{comp}_{\mathbf{v}}\mathbf{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} \quad \leftarrow \text{a number}$$

The **projection** of **u** onto **v** is the vector

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v} \quad \leftarrow \text{a vector}$$

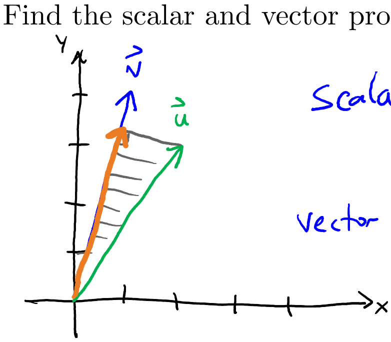
9. Draw a picture to illustrate $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{comp}_{\mathbf{v}}\mathbf{u}$.



component (or scalar projection) of $\vec{u}$ onto $\vec{v}$ is the length of the vector projection

vector projection of $\vec{u}$ onto $\vec{v}$: $\text{proj}_{\vec{v}}\vec{u}$

10. Find the scalar and vector projections of $\mathbf{u} = \langle 2, 3\rangle$ onto $\mathbf{v} = \langle 1, 4\rangle$.



Scalar projection: $\text{comp}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{\langle 2,3\rangle\cdot\langle 1,4\rangle}{\sqrt{1^2+4^2}} = \frac{2+12}{\sqrt{17}} = 3.4$ length

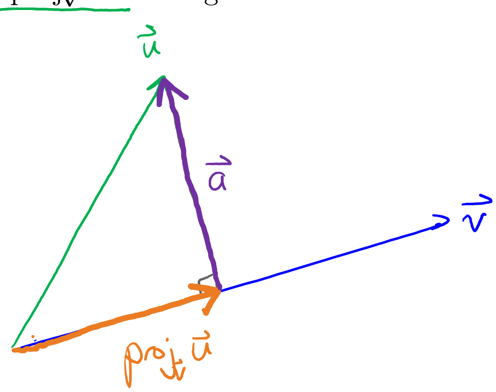
vector projection: $\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{14}{17}\langle 1,4\rangle = \left\langle \frac{14}{17}, \frac{4(14)}{17}\right\rangle = \left\langle \frac{14}{17}, \frac{56}{17}\right\rangle$ vector

11. Find the scalar and vector projections of $\mathbf{u} = 2\mathbf{j} + \mathbf{k}$ onto $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

Scalar: $\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{\langle 0,2,1\rangle\cdot\langle 2,-1,4\rangle}{\sqrt{2^2+(-1)^2+4^2}} = \frac{0-2+4}{\sqrt{4+1+16}} = \frac{2}{\sqrt{21}}$

vector: $\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{2}{21}\langle 2,-1,4\rangle = \left\langle \frac{4}{21}, \frac{-2}{21}, \frac{8}{21}\right\rangle$

12. Show that the vector $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$ is orthogonal to **v**.



$\text{proj}_{\vec{v}}\vec{u} + \vec{a} = \vec{u}$

$\boxed{\vec{a} = \vec{u} - \text{proj}_{\vec{v}}\vec{u}}$

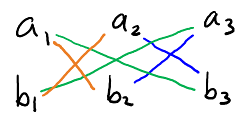
Cross Product

Definition Alert: Cross Product

The **cross product** is a calculation done with two 3-dimensional vectors. The result is another vector. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then:

$$\mathbf{a} \times \mathbf{b} \quad \text{equals} \quad \langle a_2 b_3 - a_3 b_2 \, , \, a_3 b_1 - a_1 b_3 \, , \, a_1 b_2 - a_2 b_1 \rangle.$$

$\vec{a} \times \vec{b}$



$\vec{a} \times \vec{b} = \langle a_2 b_3 - b_2 a_3, \ a_3 b_1 - a_1 b_3, \ a_1 b_2 - a_2 b_1 \rangle$

1. What is $\langle 1, 0, 0 \rangle \times \langle 0, 1, 0 \rangle$?

$$\langle 1,0,0 \rangle \times \langle 0,1,0 \rangle = \langle 0\cdot 0 - 0\cdot 1, \ 0\cdot 0 - 1\cdot 0, \ 1\cdot 1 - 0\cdot 0 \rangle$$
$$= \langle 0, 0, 1 \rangle$$

2. What is $\langle a, 0, 0 \rangle \times \langle 0, b, 0 \rangle$?

$$\langle a,0,0 \rangle \times \langle 0,b,0 \rangle = \langle 0\cdot 0 - 0 \cdot b, \ 0\cdot 0 - a \cdot 0, \ a\cdot b - 0\cdot 0 \rangle$$
$$= \langle 0, 0, ab \rangle$$

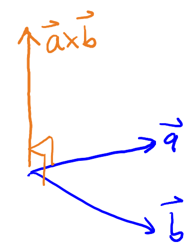
3. What is $\langle 1, 1, 1 \rangle \times \langle 2, 3, 4 \rangle$?

$$\langle 1,1,1 \rangle \times \langle 2,3,4 \rangle = \langle 1\cdot 4 - 3 \cdot 1, \ 1\cdot 2 - 1\cdot 4, \ 1\cdot 3 - 1 \cdot 2 \rangle = \langle 1, -2, 1 \rangle$$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 4 \end{matrix}$$

4. If you did #3 correctly, you should have gotten $\langle 1, -2, 1 \rangle$.
Now compute both $\langle 1, 1, 1 \rangle \cdot \langle 1, -2, 1 \rangle$ *and* $\langle 2, 3, 4 \rangle \cdot \langle 1, -2, 1 \rangle$. What do you observe?

$$\langle 1,1,1 \rangle \cdot \langle 1,-2,1 \rangle = 1 - 2 + 1 = 0$$
$$\langle 2,3,4 \rangle \cdot \langle 1,-2,1 \rangle = 2 - 6 + 4 = 0$$



5. **Make a conjecture:** How is $\mathbf{a} \times \mathbf{b}$ related to vectors $\mathbf{a}$ and $\mathbf{b}$?

$\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$ and $\vec{b}$

APPLICATION: If I want a vector perpendicular to $\vec{a}$ and $\vec{b}$, then I can use $\vec{a} \times \vec{b}$.

6. What is $\langle 2, 3, 4 \rangle \times \langle 1, 1, 1 \rangle$?

$$\langle 2,3,4 \rangle \times \langle 1,1,1 \rangle = \langle -1, 2, -1 \rangle$$

7. **Make a conjecture:** What do you think is the relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$?

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$