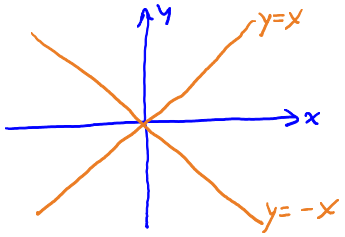
Multivariable Functions

1. Let $f(x,y) = \sqrt{x^2 - y^2}$. Note that this is a function of *two* variables, x and y!

(a) What are all x and y values such that $f(x,y) = 0$? Draw the set of these values in the (x,y)-plane.

$x^2 = y^2$ $\quad x = \pm y$

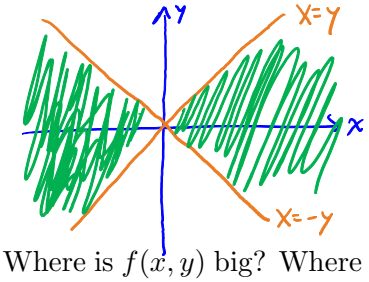
$|x| = |y|$



(b) What are all x and y values such that $f(x,y)$ is a real number? Draw the set of these values in the (x,y)-plane.

$x^2 - y^2 \geq 0$

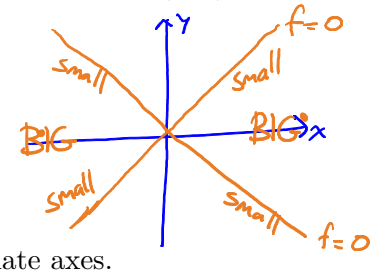
$x^2 \geq y^2$



Domain of $f(x,y) = \sqrt{x^2-y^2}$

(c) Choose some points (x,y) and compute $f(x,y)$. Where is $f(x,y)$ big? Where is $f(x,y)$ small?

$f(-100, 1) = \sqrt{(-100)^2 - 1}$

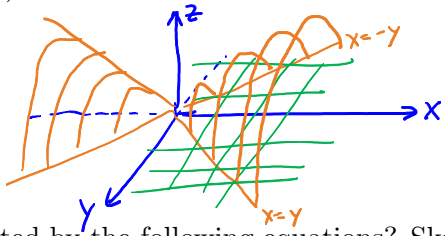


Everyone at your table can choose a different point! Then compare answers.

(d) Sketch the graph of $f(x,y)$ on the 3D coordinate axes.

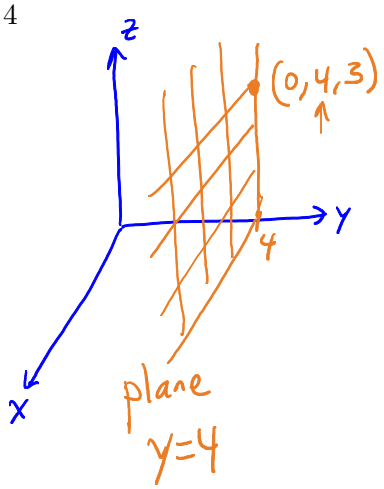
$z = f(x,y) = \sqrt{x^2-y^2}$

$z^2 = x^2 - y^2$



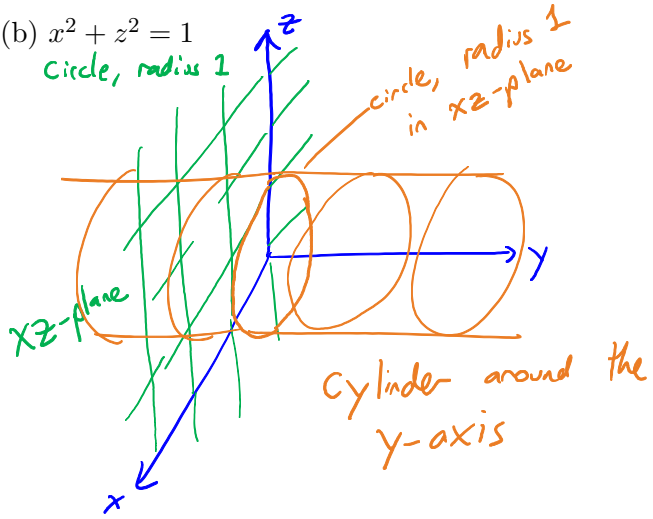
2. What 3D surfaces are represented by the following equations? Sketch these surfaces.

(a) $y = 4$



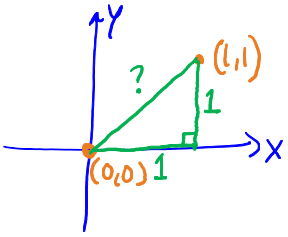
(b) $x^2 + z^2 = 1$

Circle, radius 1



3. (a) What is the distance between $(0,0)$ and $(1,1)$ in 2D?

Is there a right triangle hiding here?

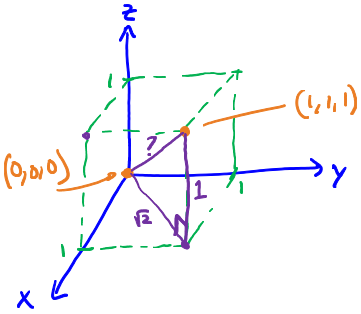


$?^2 = 1^2 + 1^2 = 2$

$? = \sqrt{2}$

(b) What is the distance between $(0,0,0)$ and $(1,1,1)$ in 3D?

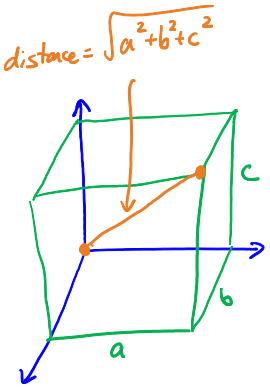
Where is the right triangle?



$?^2 = 1^2 + \sqrt{2}^2$

$?^2 = 3$

$? = \sqrt{3}$



3D Distance Formula: The distance $|PQ|$ between points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ is:

$$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

4. Consider the points $P = (1, -2, 1)$, $Q = (5, 1, 1)$, and $R = (1, 1, 1)$.

(a) Find the distance between the points P and Q.

$$|PQ| = \sqrt{(5-1)^2 + (1-(-2))^2 + (1-1)^2}$$

$$= \sqrt{4^2+3^2+0^2} = \sqrt{16+9} = \sqrt{25} = 5$$

(b) Find an equation of the sphere with radius 3 centered at P.

set of points whose distance from P is 3

Distance from (x,y,z) to $P=(1,-2,1)$ should be 3

$$\sqrt{(x-1)^2 + (y-(-2))^2 + (z-1)^2} = 3$$ ← Equation of a sphere

(c) Does R lie within the sphere of radius 2 centered at P?

Distance from R to P:

$$\sqrt{(1-1)^2 + (1-(-2))^2 + (1-1)} = \sqrt{0+9+0} = 3$$

Since this distance is greater than 2, R does not lie within the sphere of radius 2 centered at P.