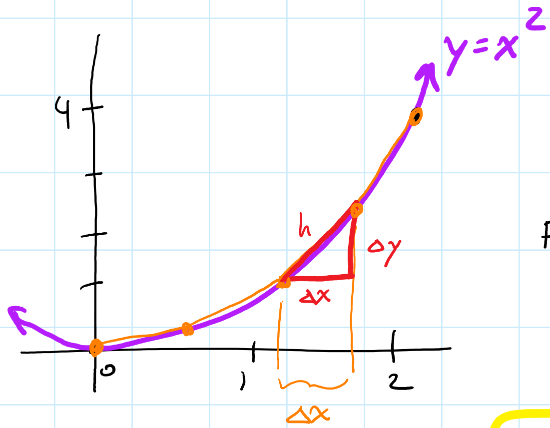
ARC LENGTH:



Find the length of the graph $y=x^2$ from $x=0$ to $x=2$.

$f(x)=x^2$

$f'(x)=2x$

Pythagorean theorem: $h=\sqrt{\Delta x^2+\Delta y^2}$

Factor out Δx^2: $h=\sqrt{\Delta x^2\left(1+\frac{\Delta y^2}{\Delta x^2}\right)} = \Delta x\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}$

slope of line segment

As $\Delta x \to 0$, slope $\frac{\Delta y}{\Delta x} \to f'(x)$.

$$\boxed{\text{"Length element"} \quad dL=\sqrt{1+f'(x)^2}\,dx}$$

Integrate this!

Arc length: $\int_0^2 \sqrt{1+f'(x)^2}\,dx = \int_0^2 \sqrt{1+(2x)^2}\,dx = \int_0^2\sqrt{1+4x^2}\,dx = \boxed{4{,}64}$

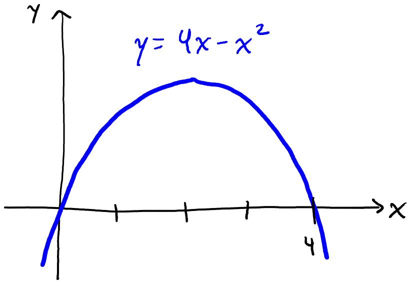
via. Wolfram Alpha

Length and Volume

The arc length of the graph of a differentiable function $f(x)$ on an interval $[a, b]$ is given by

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

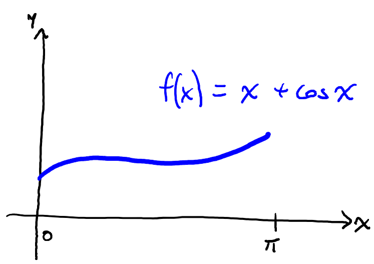
1. Find the length of the portion of the parabola $y = 4x - x^2$ that is above the x-axis. You may use technology to evaluate the integral.



$f(x) = 4x - x^2$, so $f'(x) = 4 - 2x$

$$\text{Length} = \int_0^4 \sqrt{1 + (4-2x)^2}\, dx = 9.2936$$

2. Find the length of the graph of $y = x + \cos(x)$ for $0 \le x \le \pi$. Use technology to evaluate the integral.

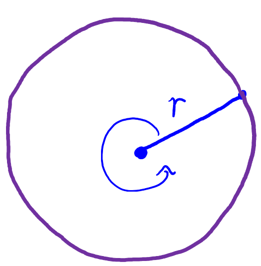


$f'(x) = 1 - \sin x$

$$\text{Length} = \int_0^\pi \sqrt{1 + (1 - \sin x)^2}\, dx = 3.45886$$

3. It's time to do some drawing! (Use another sheet of paper.)

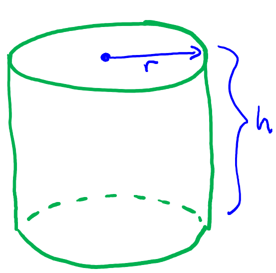
(a) Draw a single line segment. What shape do you get as you rotate the segment 360 degrees (2π radians) while keeping one end fixed? What is the area of this shape?



You get a circle!

Area: πr^2

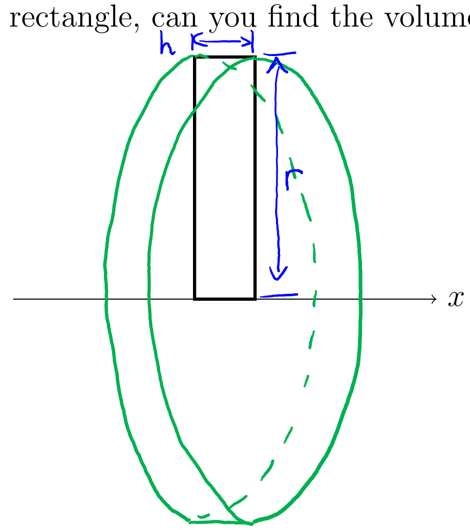
(b) Draw the best picture of a cylinder that you can. Label the cylinder so that the radius of the base is r and the height is h.



(c) Do you remember how to compute the *volume* of a cylinder? What is the formula? Try to explain to the people at your table *why* your formula works for the volume of a cylinder.

Volume of a cylinder: $\pi r^2 h$

(d) Here is a rectangle. We want to rotate the rectangle 360 degrees around the x-axis, producing something in 3-D. Try drawing the resulting 3-D shape. If you know the height and width of the rectangle, can you find the volume of your 3-D shape?

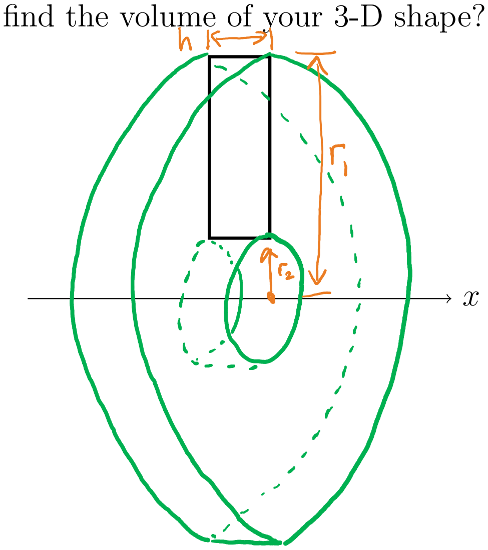


Volume $= \pi r^2 h$

(e) Here is another rectangle. Again we want to rotate the rectangle 360 degrees around the x-axis. What 3-D shape do you get now? If you know the height and width of the rectangle, can you find the volume of your 3-D shape?

Volume:

$\pi r_1^2 h - \pi r_2^2 h$



4. (a) Carefully draw the graph of $f(x) = x^2$ and the region bounded by $f(x)$, $x = 2$, $x = 3$, and the x-axis. Draw some sample rectangles of the type that you would use to approximate the area of this region.

(b) Rotate one of your sample rectangles around the x-axis to get a 3-D shape. What is the formula for the volume of this 3-D shape?

(c) Now suppose the entire region from part (a) is rotated around the x-axis. What is the volume of the resulting shape? *Hint*: integrate the volume formula you found in part (b)!

to be continued...

5. (a) Carefully draw the graph of $f(x) = x$, $f(x) = x^3$, and the region bounded by both graphs. Draw some sample rectangles of the type that you would use to approximate the area of this region.

(b) Rotate one of your sample rectangles around the x-axis to get a 3-D shape. What is the formula for the volume of this 3-D shape?

(c) Now suppose the entire region from part (a) is rotated around the x-axis. What is the volume of the resulting shape? *Hint*: integrate the volume formula you found in part (b)!

6. Consider again the region bounded by the graphs of $f(x) = x$ and $f(x) = x^3$.

(a) Find the volume when this region is rotated around the line $y = -1$.

(b) Find the volume when this region is rotated around the line $y = 3$.

(c) Find the volume when this region is rotated around the y-axis.