Written Homework 9

Math 126

Solve each of the following problems. Write your solutions clearly and neatly on separate paper, explaining your reasoning with complete sentences. Submit your work either in class or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Monday, October 21**.

1. Let's return to the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Consider the following grouping of terms of the harmonic series:

$$1 + \left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{16}\right) + \cdots$$

(a) Show that the sum of each group of fractions is more than $\frac{1}{2}$.

(b) Explain why this shows that the harmonic series does not converge.

2. Although the harmonic series does not converge, the partial sums grow very, *very* slowly.

(a) Use a comparison with $\displaystyle\int_1^b \frac{1}{x}\, dx$ to explain why $\displaystyle\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} < \ln(n)$.

(b) If a computer could add up one billion terms of the harmonic series each second, estimate what the sum would be after one year.

Problem 3 has been moved to Written Homework 10.