

Written Homework 10

Math 126

Solve each of the following problems. Write your solutions clearly and neatly on separate paper, explaining your reasoning with complete sentences. Submit your work either in class or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Wednesday, October 23**.

1. We have seen that if the terms of a series approach zero, the series might or might not converge.

(a) Give an example of a *convergent* series $\sum_{n=1}^{\infty} a_n$ such that a_n approaches 0 as $n \rightarrow \infty$.

(b) Give an example of a *divergent* series $\sum_{n=1}^{\infty} a_n$ such that a_n approaches 0 as $n \rightarrow \infty$.

2. Give specific example of a series $\sum_{n=1}^{\infty} a_n$ that converges to 15.

3. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{nx^n}{3^n}$. Clearly show what tests you use and how you arrive at your conclusion.

4. Consider the series $2 + 2 \cdot 3x + 3 \cdot 4x^2 + 4 \cdot 5x^3 + 5 \cdot 6x^4 + 6 \cdot 7x^5 + \dots$. What function does this series converge to? For what values of x does it converge? (*Hint*: Remember the series we looked at in class on October 18.)

5. Suppose that you apply the ratio test to the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n}.$$

(a) For what values of x does the ratio test show that the series converges?

(b) For what values of x does the ratio test show that the series diverges?

(c) For what values of x is the ratio test inconclusive?